The Modified Expectations Equity Curve

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Abstract

The Modified Expectations Equity Curve is a technique that adjusts historical strategy performance to account for the potential of out of sample signal weakening. In sample returns are adjusted to transform their Sharpe ratios based on two user selected parameters. The resulting adjusted return stream may prove to be a better forecast of future returns and thus more useful for determining optimal strategy allocation weightings.

1. Introduction

The future is wrought with uncertainty. Market conditions will change as periods of calm and tumult oscillate. And for the quantitative investment manager, historical return vectors and covariance matrices will shift in the future as well. In this paper I introduce a new technique that alters historical returns to adapt for changes in out of sample returns, allowing for more robust allocation by strategy and more realistic out of sample return expectations.

2. Strategy excess returns over time

Market efficiency is like a rubber band. As strategies produce abnormal returns, they are likely to draw additional interest and capital, pushing expected returns closer to zero over time. Whether fixed income convergence trades, high frequency equity market making, or commodity volatility arbitrage, outsized historical gains will close over time as inefficiencies are identified and new capital is deployed. Consistent with the rubber band analogy, strategies that stretch the rubber band tighter with hefty returns will snap back quicker than strategies with only slightly excess returns. The larger the risk adjusted profitability, the quicker attention will be garnered and inefficiencies will close.

Farmer and Skouras (2011) present a model estimating the time it would take the marketplace to detect and exploit inefficiencies for strategies based on induction from past data. They estimate that the minimum time before a strategy would attract enough capital to exploit that strategy is $t=8/SR^2$ where t is time in years and SR is the strategy's Sharpe ratio. Their model suggests that a strategy with a Sharpe ratio of 1.00 might take eight years of existence before capital is employed to diminish its returns. Separately, a strategy with Sharpe ratio of 2.00 might only take two years before such edge was exploited.

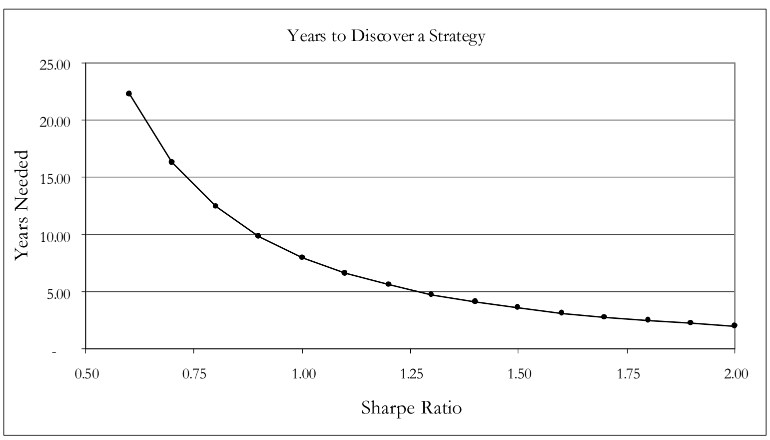

Figure 1. Years needed to detect and exploit a strategy based on induction from historical data (citing Farmer and Skouras' formula)

It would be rare for any strategy to be exploited to zero return immediately. A more likely outcome is that inefficiencies are discovered by market participants progressively over time, slowly eroding excess returns. I suggest in Kestner (2003) that the decrease in returns is more likely to follow an exponential decay instead of a more abrupt closing of market inefficiencies.

This tendency for returns to diminish over time creates a quandary for the quantitative trader focusing on systematic strategies. Historical returns, volatilities, and correlations are all used as inputs to create an optimized portfolio that maximizes a certain performance criteria. If expected returns are likely to decrease in the future compared to historical returns, the resulting portfolio optimization using historical returns is not optimal. If taken at face value, the resulting portfolio's historical return will lead to grossly unrealistic expectations. More dangerously, the allocation process can cause added harm as the best performing strategies receiving the largest risk allocations are likely to see the largest decline in out of sample performance as competition arbitrages away excess returns.

A better way to allocate risk by strategy and also estimate out of sample performance is to scale expected performance on a strategy by strategy basis, penalizing the best performing strategies more than the rest. The Modified Expectation Equity Curve (MEEC) completes this task by compressing historical returns towards a user selected baseline Sharpe ratio S by a compression percentage C.

3. Modified Expectations Equity Curve (MEEC) calculations

The MEEC calculations start by computing historical Sharpe ratios of individual strategies.

$$SR = \frac{\bar{r}}{\sigma_r}$$

The MEEC compresses the historical return of each strategy to reduce the in sample Sharpe ratio by a user defined percentage between the historical Sharpe ratio and the user defined baseline Sharpe ratio of *S*. The MEEC will penalize strategies with higher in sample Sharpe ratios more than those with lower Sharpe ratios.

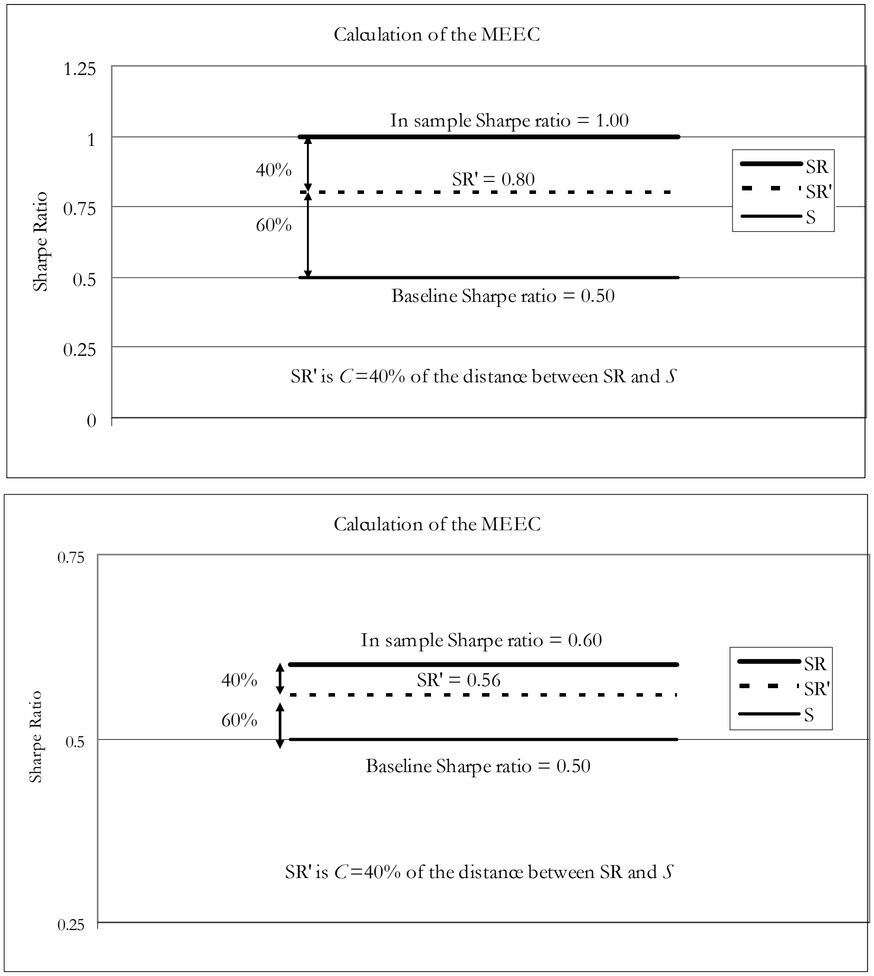

Figure 2a and 2b. The MEEC calculations at work. The adjusted Sharpe ratio is calculated by reducing the in sample Sharpe ratio by a compression factor *C* to a baseline Sharpe ratio *S*.

Figures 2a and 2b detail the MEEC at work. Suppose we have a strategy with Sharpe ratio equal to 1.00. The user has chosen a baseline Sharpe ratio *S* of 0.50 and a compression percentage *C* of 40%. The MEEC is created by reducing the strategy's Sharpe ratio by *C*=40% of the distance between the actual Sharpe ratio SR=1.00 and the baseline *S*=0.50. The new adjusted Sharpe ratio (SR´) is calculated to be 0.80.

$$SR' = SR - (C \cdot SR) + (C \cdot S)$$

$$SR' = 1.00 - (40\% \cdot 1.00) + (40\% \cdot 0.50) = 0.80$$

where S is the user selected Sharpe ratio floor and C is the user selected compression percentage

A similar calculation is completed in Figure 2b, only starting with a strategy with Sharpe ratio 0.60. In this example, the adjusted SR´ is 0.56. As long as the user selected baseline S is greater than zero, higher Sharpe ratio strategies will be penalized more than lower Sharpe ratio strategies. Also, the S parameter should be selected no higher than the lowest strategy Sharpe ratio, otherwise the MEEC could have the unintended consequence of improving historical returns.

Once the adjusted Sharpe ratios (SR´) have been calculated, the original return streams are reduced by a fixed return amount per strategy[1]. The result is a lower Sharpe ratio equal to SR´ calculated above.

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[1] If the historical and baseline Sharpe ratios have been adjusted for periodicity (e.g. multiplied by $\sqrt{12}$ to calculate annualized Sharpe ratios using monthly return series), the adjusted return formula needs to incorporate this scaling multiplier. Using annualized Sharpe ratios from monthly return data, the adjusted return formula would be:

$$r'_{i,t} = r_{i,t} - \bar{r}_i \left(\frac{C \cdot \frac{SR}{\sqrt{12}} - C \cdot \frac{S}{\sqrt{12}}}{\frac{SR}{\sqrt{12}}} \right)$$

$$r'_{i,t} = r_{i,t} - \bar{r}_i \left(\frac{C \cdot SR - C \cdot S}{SR} \right)$$

4. Example in practice: diversified strategies

The next two sections of this paper detail real world examples of the Modified Expectations Equity Curve in practice. In this section, I use the MEEC on three diverse strategies to show the difference of mean variance optimization on (1) the raw return streams and (2) the adjusted return streams using the MEEC technique.

I start with weekly returns of three strategies: the DB Trends Excess Return Index (Bloomberg: DBTRDUSX), the ISE Natural Gas Futures Spread Index (Bloomberg: GYY), and the CBOE VIX Premium Strategy Index (Bloomberg: VPD). Each of the three strategies attempts to exploit very different market opportunities including movements in Eurodollar futures, spread positions in natural gas futures, and short positions in VIX futures. The choice of these three indices was mildly random and should not be taken as a sign of a complete research effort. All three display positive returns of varying degrees over the past seven years – perfect for a test of the MEEC.

Weekly data was compiled between 2006 and 2012 and descriptive statistics of the three strategies are detailed in Table 1 below. Annualized returns in excess of 1 month LIBOR run from 6% to 12% and volatilities between 11% and 22%. Optimized portfolios are created in two ways. First, each strategy's weight is selected by maximizing the portfolio's Sharpe ratio. Second, each strategy's Modified Expectations Equity Curve is calculated using a baseline Sharpe ratio of S=0.30 and compression percentage C=50%. Using each strategy's MEEC, another portfolio is created again by selecting each strategy's weight to maximize the portfolio's Sharpe ratio.

Table 1. Descriptive statistics of DB Trends, ISE Natural Gas Futures Spread Index, and the CBOE VIX Premium Strategy Index.

	DB Trends	ISE Natural Gas	VIX Premium Strategy
Annualized excess return	12.73%	11.63%	6.66%
Volatility	11.96%	14.66%	22.63%
Sharpe ratio	1.06	0.79	0.29
User defined S	0.30	0.30	0.30
User defined C	50%	50%	50%
SR'	0.68	0.55	0.30

The equity curves, strategy weights, and portfolio statistics are detailed in Table 2 and Figure 3. Note how the MEEC based strategy allocation increases weights for lower Sharpe ratio strategies such as VPD and decreases allocation to the higher Sharpe ratio strategies of DBTRDUSX and GYY. Also, the annualized portfolio excess return decreases from 11.57% for raw returns to 7.88% (a decrease of 32%) for the MEEC portfolio – a likely much better estimate of out of sample expected returns.

Table 2. Descriptive statistics for the two optimized portfolios.

	Raw return portfolio	MEEC Portfolio	Difference
DB Trends Weight	55.42%	52.46%	-2.96%
ISE Natural Gas Weight	31.17%	31.04%	-0.13%
VIX Premium Weight	13.40%	16.49%	3.09%
Annualized excess return	11.57%	7.88%	-370bps
Volatility	8.16%	8.07%	-0.09%
Sharpe ratio	1.42	0.98	-0.44

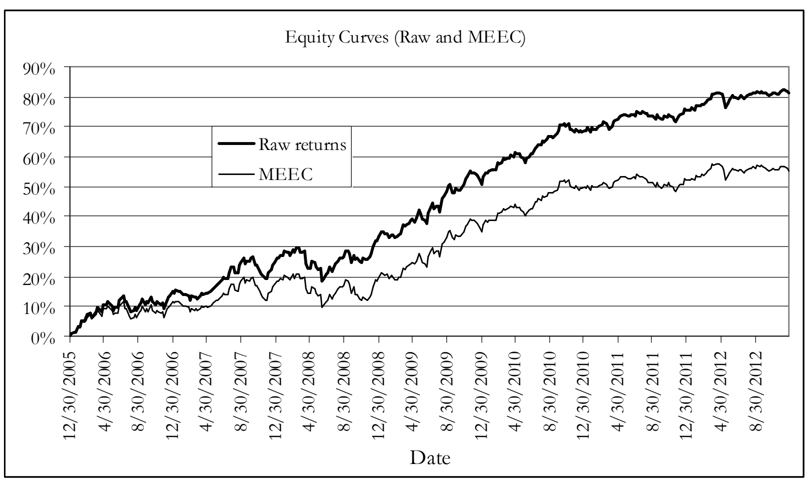

Figure 3. Equity curves for (1) the portfolio optimized using in sample raw returns and (2) the portfolio optimized using individual Modified Expectation Equity Curves.

5. Example in practice: risk parity at historical reward to risk levels

Another clever use of the MEEC is to evaluate risk parity strategies, but adjust expected reward to risk levels of each asset class towards long run historical averages.

While risk parity investing has gained traction with many investors in recent years, detractors of the risk allocation method cite the high weighting of lower volatility assets such as interest rate duration after a long period of above average returns and historically low starting levels. One way to normalize the bond outperformance/stock underperformance over the past decade is to use the Modified Expectations Equity Curve technique to adjust returns over the past 10 years – a timeframe where bond performance dominated. Using the new MEEC equity curves, we can glean insight into how risk parity strategies might perform during periods outside of the recent bond bull market.

A *Daily Observations* piece written by Bridgewater Associates in 2004 looked at long run returns by asset class after scaling risk to levels generally seen in equities using leverage. Versus long run equity volatility of 15% annualized, excess returns for various sectors of equities, rates, and credit yielded between four and six percent per annum. Taking the midpoint across all asset classes and assuming a 5% annualized excess return on 15% annualized risk leads to a Sharpe ratio of 0.33. Making the perhaps naïve assumption that long run expectations across all asset classes yield Sharpe ratios of 0.33, we can use the MEEC technique to push returns towards that

0.33 Sharpe ratio by selecting a baseline Sharpe *S* of 0.33 and a compression percentage *C* of 100%.

To test the effect, we simulate a very simple risk parity portfolio consisting of three assets: the S&P 500 Total Return Index (Bloomberg: SPTR), Barclay's US Aggregate Total Return Index (Bloomberg: LBUSTRUU), and the Dow Jones-UBS Commodity Total Return Index (Bloomberg: DJUBSTR). Each asset's monthly weight is selected to be in inversely proportional to the trailing 200 day volatility. The portfolio is rebalanced monthly spanning the years 2000 through 2012 and is fully invested without use of leverage.

$$w_i = \frac{\left(\frac{1}{\sigma_i}\right)}{\sum_{k=1}^{N}\left(\frac{1}{\sigma_k}\right)}$$

The raw risk parity portfolio yields annualized excess returns of 3.27% and volatility of 4.88%. When the MEEC adjustment is made using *S*=0.33 and *C*=100% (forcing historical performance to long run means of SR=0.33), returns fall by 23.5% to 2.50% per annum (a decline of 77bps). This simple exercise suggests that if reward to risk performance over the last 13 years had been in-line with long run results, the risk parity portfolio would in fact have seen lower returns. Such analysis may help evaluate risk parity portfolio expected returns under more "normal" regimes.

Table 3. Descriptive statistics of the S&P 500 Total Return Index, Barclay's US Aggregate Total Return Index, and Dow Jones-UBS Commodity Total Return Index.

	S&P 500 Total Return	Barclay's US Aggregate	DJ-UBS Commodity
Annualized excess return	0.43%	3.68%	4.37%
Volatility	16.04%	3.56%	17.50%
Sharpe ratio	0.03	1.03	0.25
User defined S	0.33	0.33	0.33
User defined C	1.00	1.00	1.00
SR'	0.33	0.33	0.33

Table 4. Descriptive statistics for (1) the raw return portfolio and (2) the MEEC (S=0.33, C=100%) adjusted portfolio.

	Raw return portfolio	MEEC Portfolio	Difference
Annualized excess return	3.27%	2.50%	-77bps/-23.5%
Volatility	4.88%	4.87%	-0.01%
Sharpe ratio	0.67	0.51	-0.16

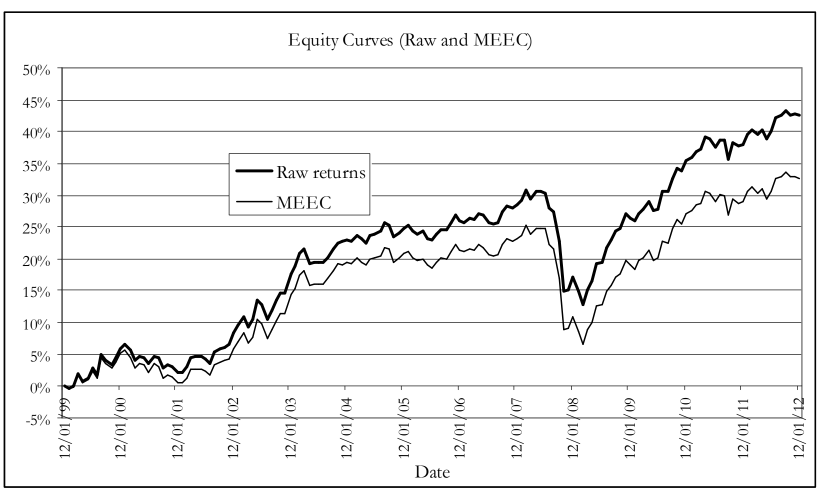

Figure 4. Equity curves for (1) the risk parity portfolio using raw returns and (2) the risk parity portfolio using individual MEEC equity curves.

6. Conclusion

The Modified Expectations Equity Curve is a powerful tool that can be used for a wide range of tasks. Its core function is to moderate in sample performance on a strategy by strategy basis based on two user specified parameters. Because capital is continually flowing to push market inefficiencies towards zero, these lowered expectations can provide a more accurate picture of out of sample performance as well as aid in allocating risk by strategy. While MEEC results will be shaped largely by the user's choice of the baseline Sharpe ratio (S) and compression (C) parameters, this flexibility makes the tool valuable to market participants having wide ranging views on prospective performance. Further research may yield more concrete analysis of how quickly out of sample performance deteriorates, allowing for more accurate decisions of baseline Sharpe ratios (S) and compression percentages (C).

References

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