A More Quantitative Approach to “A Quantitative Approach to Tactical Asset Allocation”

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Abstract: Faber (2009), one of the most downloaded investment papers on SSRN, details a Tactical Asset Allocation investment strategy that aims to take advantage of periods where returns from some asset classes are below average and volatility is much higher. In other words, his strategy takes advantage of different market regimes. Though his exact strategy may not coincide with the investment goals of financial institutions due to the binary investment decisions in Faber’s strategy, the advantages of investing dependent on the regimes of different asset classes are important enough that institutions should not avoid Tactical Asset Allocation.

This paper confirms Faber’s approach that taking advantage of economic cycles can significantly improve risk-adjusted returns. There are significant improvements to risk-adjusted returns by incorporating conditional expected returns and standard deviations dependent on the state of the regime. These forecasts are created both using simple 10-month moving averages and with more complex Markov regime-switching methods. Finally, a variety of extensions, including adjusting maximum leverage, risk aversion coefficients, and tracking error bounds, can improve performance of the basic strategy. Furthermore, taking into account the cyclicality and idiosyncratic momentum of various sub-indices to Faber’s original asset classes produces even stronger improvements to risk-adjusted returns.

Due to the nature of the tools used, these strategies tend to match benchmark returns more closely than Faber’s strategy does during market corrections. Hence, this strategy does not offer to protection on the downside to the degree that Faber’s does. These strategies over-weight cyclical assets during bull markets and under-weight them during bear markets, just not to the degree that Faber’s strategy under-weights them.

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Section 1: Introduction

Faber (2009) is one of the most popular investment articles downloaded on SSRN[2]. Faber details a Tactical Asset Allocation (TAA) strategy that invests in five asset classes, U.S. stocks, foreign stocks, U.S. long-term bonds, commodities, and U.S. REITs[3] depending on whether they are above or below their 10 month (equivalent to 200 day if daily data is used) moving average. If the asset class is above the 10 month moving average, the strategy invests 20% of the portfolio in the asset class; otherwise, the strategy invests 20% in a short-term risk-free security[4]. While the strategy will not avoid a sharp downturn like the crash in 1987, it can help avoid prolonged bear markets while maintaining gains on the upside. Since 1970, this strategy would have significantly outperformed an equally weighted portfolio of these five asset classes with a higher Sharpe ratio and lower drawdowns.

Faber's strategy, even in his own firm Cambria Investment Management, is generally not implemented as the paper describes without making adjustments. For a financial institution, such as an endowment or pension fund, the biggest concern will likely be the tracking error of Faber's strategy. By making a binary decision to invest in an asset class or avoid it, there can be significant deviations from how the benchmark performs, resulting in a tracking error ex post of around 7% annually since the 1970s. An endowment may be less concerned about the deviations when the market is falling swiftly, but when markets rebound, this strategy tends to stay in cash until a significant rally has already occurred. For instance, during the recession from 2007-2009, this strategy was timely in exiting the U.S. and foreign stock markets. However, after the market began to turn around in March 2009, there was a significant delay before the 10 month moving average strategy would begin to invest in these asset classes, losing out on a nearly 30% recovery in the S&P500. Since the recovery from an economic crisis is historically one of the strongest periods of gains for equity markets, an institution would likely not tolerate such strong underperformance.

Nevertheless, there is a very important truth imbedded in Faber's strategy: economic cycles can make certain asset classes more attractive than others. Economic cycles generally

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[2] It has had 47,690 downloads, making it the 4th most frequently downloaded paper on SSRN as of March 14th, 2010.

[3] Faber uses total return indices for the S&P500, MSCI EAFE, Global Financial Database's 10-year treasury return index, the GSCI index, and NAREIT's index for each of the above asset classes.

[4] 3 month U.S. T-bills are used throughout the paper as the risk-free rate.

have a myriad of causes, but so long as they occur and have differing implications for different asset classes, investment strategies should take advantage of these cycles. This paper adds to Faber's strategy by implementing it in a context that would be more relevant to a financial institution, such as by performing optimizations and by utilizing ex ante tracking error bounds. Further, forecasts for expected returns and standard deviations used in active portfolio construction, including simple conditional values dependent on whether an asset class is above or below a moving average and Markov regime-switching conditional values, can help mirror Faber's strategy. However, there are nearly boundless ways to compute forecasts for expected returns or covariance matrices. Finally, this study examines various extensions that can be used to either improve returns or reduce the risk of the strategy, such as adjusting leverage depending on how wide or narrow the Baa-Treasury spread is.

This paper confirms Faber's approach that taking advantage of economic cycles can significantly improve risk-adjusted returns. There are significant improvements to risk-adjusted returns by incorporating conditional expected returns and standard deviations dependent on the state of the regime. These forecasts are created both using simple 10-month moving averages and with more complex Markov regime-switching methods. Finally, a variety of extensions, including adjusting maximum leverage, risk aversion coefficients, and tracking error bounds, can improve performance of the basic strategy. Furthermore, taking into account the cyclicality and idiosyncratic momentum of various sub-indices to Faber's original asset classes produces the strongest improvement to risk-adjusted returns.

Due to the nature of the tools used, these strategies tend to match benchmark returns more closely than Faber's strategy does during market corrections. Hence, this strategy does not offer to protection on the downside to the degree that Faber's does. These strategies over-weight cyclical assets during bull markets and under-weight them during bear markets, just not to the degree that Faber's strategy under-weights them.

Section 2: Theory/Motivation

Though the popularity of the strategy in Faber (2009) is due to its simplicity, why it works is more interesting. His strategy essentially boils down to getting out of markets fully when they are in bear markets, but then participating in the gains when markets trend. The biggest benefit comes from the reduction in risk, not necessarily producing outsized gains.

However, if investors could simply get out of the market at the beginning of a bear market and generate higher Sharpe ratios, why don't all investors do this?

Attempting to answer this question, Marmi et al. (2009) investigate the outperformance of Faber's strategy by simulating S&P500 returns and then constructing bootstrap portfolios. Their paper then checks whether the outperformance of Faber's 10 month moving average strategy is statistically significant relative to the bootstrap results. Marmi et al. conclude that "the outperformance of the strategy is compatible with the statistical variability of the historical returns." They also reject the idea that Faber's strategy serves as a counter-example to market efficiency. While they do utilize a GARCH model to account for time-varying volatility, they do not explicitly account for bear and bull markets. In many ways a better model for market returns is from a Markov regime-switching model. This type of model allows specifically for periods similar to bear markets where average returns are lower and the variance is higher and boom markets where average returns are higher. The model also incorporates a probability that one regime will transition to another so that bull and bear markets can last for different periods. Thus the conditional mean from the model output, depending on the probability of being in one regime or another, tends to matter most as an input into portfolio construction. Faber's method might have been confirmed if the authors had simulated returns based on a regime-switching framework instead.

Regime-switching models occupy a significant place in academic literature. Starting from the seminal paper by Hamilton (1989), authors have investigated how different macroeconomic variables behave in regimes. There is substantial evidence of regime-switching in interest rates, exchange rates, and stock market returns. Ang and Bakaert (2002) provide the most relevant framework for using a regime-switching conditional expected returns and standard deviations in asset allocation. They look at how regimes affect stock returns, bond returns, and a risk-free asset and then adjust asset allocations depending on the conditional estimate of what regime it is. Extending Ang and Bakaert's regime-switching multivariate estimation to five asset classes would incorporate regimes into a framework like Faber's while accounting for regime-switching in correlations between assets. However, allowing for changing correlations greatly expands the number of parameters that are needed to estimate the model[5]. By ignoring the possibility of

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[5] This also greatly extends the length of time required to estimate the model, precluding even an in-sample estimate. The in-sample model with regime-switching correlations tends to take over six hours to

regime-switching correlations, there is little advantage in following Ang and Bakaert's framework exactly. Hence, the best approach is to adopt a framework that most closely matches Faber's strategy.

From the perspective of this paper, the most interesting item about regime-switching is how it relates to the 10 month moving average strategy proposed by Faber. Faber would probably argue that his strategy results in outperformance because there are bull and bear markets, which is essentially what the regime-switching models imply when two regimes are used. Faber is essentially capturing the regime-switching nature of asset returns with the 10 month moving average. The framework from Ang and Bakaert assumes that there is one regime that impacts all markets, but a regime-switching strategy most comparable to Faber's would allow for different asset classes to have their own regime.

One alternative to Ang and Bakaert is to include multiple regimes, such as a low returns regime, a high returns regime with high inflation, and a high returns regime with disinflation, in estimating conditional expected returns and standard deviations. Unfortunately, this will greatly increase the number of parameters required, particularly in a multivariate setting. A better alternative is to estimate one regime-switching equation for each asset class, allowing for the possibility that the regimes impacting each asset class may not occur at the same time. This method disregards changing correlations between asset classes that the multivariate method theoretically can incorporate, but most closely resembles the spirit of Faber's paper[6].

Regime-switching in many of these asset classes is most likely driven by underlying economic factors. For instance, stock markets typically fall during recessions and central banks lower short-term interest rates during recessions which tends to boost bond returns. The investor may not be able to time when markets fall or rise exactly, but so long as she can get an estimate of the probability that markets are in one regime or another, she can compute conditional expected returns and standard deviations necessary for portfolio construction. All TAA requires is that markets are regime-switching. So long as there are economic cycles and regime-switching markets, there will be room to adjust asset allocations to take advantage of this fact.

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estimate (I never actually ran it until it fully calculated), so this criticism might be over-exaggerated until computing power or the speed of the Matlab add-in used to estimate these improves.

[6] Another advantage is that it is significantly faster to estimate a few number of equations many times than estimate a large set of equations.

The motivation of this paper is to use more quantitative tools to take advantage of these regimes. Most institutions, like pension funds or endowments, would not utilize as simple a strategy as the one popularized in Faber's article. The most important reason is that the tracking error of his strategy relative to the equally-weighted benchmark is quite high, around 7% ex post, due to the binary decisions the strategy makes[7]. Faber's approach is most appropriate for individuals who are less concerned by large tracking errors. Faber has noted that his firm does not implement his strategy exactly. Significant changes need to be adopted before a strategy based on his ideas would gain wider acceptance at asset management firms, endowments, and pension funds.

The fundamental insight of Faber's strategy is that returns are different depending on what regime it is. By breaking the strategy into two components, the benchmark (or passive) portfolio and the active portfolio, it becomes more obvious how quantitative methods can improve this strategy. In Faber's strategy the relevant benchmark is an equally weighted portfolio of five asset classes, but there is no reason beyond simplicity is given as to why this benchmark should be chosen over some alternative.

The active portfolio is the one most relevant from the perspective of the active investment manager. For each asset, Faber's active weights are essentially 0% if the market is above the 10 month moving average and -20% if below[8]. Creating forecasts of expected returns and the covariance matrix using more quantitative methods and then implementing them in portfolio construction and optimization can provide more risk controls and allow additional flexibility. So long as the forecasted returns are an improvement over naively taking historical returns, this strategy will likely improve risk-adjusted returns.

Some managers might eschew an approach that relies heavily on quantitative methods. However, relative to the standard methods taught in business schools, paying attention to economic regimes can provide a significant amount of flexibility. First, in periods where recent returns and standard deviations differ substantially from historical levels, conditional expected returns or standard deviations are likely generate more appropriate portfolio weights than historical estimates will. Furthermore, while for the sake of simplicity I assume the correlation

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[7] By comparison, when using a 3% tracking error bound, risk aversion coefficient of 3, and maximum leverage of 25%, the moving average strategy has an ex-post tracking error of 2.5% and the Markov regime-switching strategy has a tracking error of 3%.

[8] Recall benchmark weights are 20% for each asset class and Faber's strategy goes to cash.

structure is constant between regimes, there are methods that can better take into account changing correlations in a less time-consuming fashion. In addition, quantitative methods can provide a systemic way to test how a strategy has performed in the past, using that knowledge as insights to estimate future performance. While past performance is in no way indicative of future results, backtesting can help reveal the benefits or shortcomings of a strategy. Finally, quantitative methods take the emotion out of the asset allocation decision. Frequently there are investors who are worried about a stock decline in the U.S. and may invest large portions of their portfolio in foreign stocks or gold. However, only in certain circumstances may it make sense to do that and quantitative methods can be better to reveal these cases.

From an economic perspective, there is a concern that the returns to this type of strategy would be diminished if the average investor allocated their capital in this fashion. There's no doubt that if forecasted returns and covariance matrices match the historical ones, then there will be little incentive to adopt a TAA strategy. However, since the strategy is driven largely by the cyclicality of macroeconomic variables, there is little reason to fear that forecasted moments would match historical moments unless the business cycle is conquered and there is only one regime.

That being said, there are some who would argue that if households and entrepreneurs follow rational expectations, then there should be no opportunity to profit from these deviations. In some respect, systemic mistakes in the pricing of asset classes are required for TAA strategies to outperform their benchmarks[9]. However, two factors will likely prevent this strategy from losing alpha in the future: relative pricing is more important in markets than pricing between asset classes and economic cycles are a fixture of the economy. First, securities are often priced relative to other securities. Except for some hedge funds focusing on Global Macro and other tactical strategies, the bulk of pension fund and endowment capital is allocated on the basis of strategic, rather than tactical, asset allocation[10]. Indeed, nearly the entire U.S. mutual fund industry is organized to sell products for particular slices of styles or sectors of asset classes. So if there are large mispricings between asset classes, it can take time for households or funds to adjust allocations to take advantage of this fact, whereas mispricings between individual stocks or different commodities contracts are generally more quickly noticed by these

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[9] And is not evidence that they have worked not count for something against this argument?

[10] The situation is similar with individuals on average who (at best) use programs from IRA or 401(k) websites to help determine asset allocation.

funds and analysts specializing in styles or sectors. Second, economic cycles in recent decades have been driven to a large extent by the cyclicality of credit expansion. However, it is unclear what prices should have been in the absence of credit expansion. Without the prices from the counterfactual situation where there is no economic cycle, it becomes very difficult for marginal entrepreneurs to know just how bad their investments truly are and how to mitigate the risk. This helps the economic cycle to continue gathering momentum until the credit expansion begins to slow. Nevertheless, even if economic cycles do diminish in volatility, this does not necessarily imply that asset returns will not become over- or under-priced. For instance, TAA strategies performed well during the internet bubble and crash, even though U.S. GDP growth barely declined.

Finally, there is also a fear that if all investors allocated money in this fashion then there could be a situation like 1987 where all investors are selling at the same time. First, this strategy adopts a longer-term perspective than portfolio insurance and can be adjusted in all kinds of ways so that it may not produce the same portfolio weights for all investors. Further, the strategy in this paper, relative to Faber's, is typically much less aggressive in adjusting weights. In addition, many firms offer TAA mandates, but historically it has only been a small percentage of assets under management. Finally, academics generally teach that this kind of strategy fails due to the lack of evidence that market timing works or that investors should invest in anything beyond index funds[11]. Without widespread adoption of TAA srategies, this is of little concern.

Section 3: Choice of Benchmarks

Though an active manager may be evaluated against a benchmark, the choice of the benchmark is critical for the investor's total return. Three factors can assist in determining the choice of benchmarks: portfolio statistics like the Sharpe ratio, whether the benchmark incorporates historical returns or covariance matrices, and whether it incorporates economic significance of the underlying assets. Some of the options for benchmarks might incorporate the historical distributions of the asset classes or might incorporate economic significance. However, few incorporate both. This paper considers several different methods of creating benchmarks

[11] I generally agree that security selection within asset classes or the breakdown of an asset class into sub-indices generally adds little value on average given the costs. However, the evidence I present in the paper suggests that substantial value can be added through TAA, especially since an investor can invest in the index of each individual asset class.

since different managers with different objectives may find different methods more or less attractive.

Unless mentioned otherwise, covariance matrices used in portfolio construction are calculated using weighted covariance matrices. Weights are chosen so that the most recent half of the data receives a 2/3rds weight.

First, Faber (2009) uses an equal-weighted portfolio of the five asset classes as his benchmark. The benefit of equal-weighted portfolios is that it is simple and easy to explain. However, using an equal-weighted portfolio has no relation to the economic significance of the asset classes or any of the moments of the return distributions for the assets.

The most common alternative to the equal-weighed benchmark is to use weights based on the market capitalization (market-cap) of the asset classes[12]. The benefit of market-cap weights is that they reflect the underlying economic significance of each asset class. However, this strategy requires market-cap weights for commodities, which do not have a market capitalization. One method utilized by Izorek (2006) is to use the market-cap weights for all the other assets and adjust the weight on the share of commodities share until utility is maximized in a mean-variance framework or until the Sharpe ratio is maximized. Unfortunately, the data for many of these series begins in 1970 and if the 1970-1975 periods serve as a training period, then the share for commodities that maximizes the Sharpe ratio or utility is particularly high, usually greater than 50%, in the early years since stocks perform poorly and commodities perform well.

There are numerous possible solutions to this dilemma involving more or less complexity. Alternately, I ultimately decided to simply calculate the Sharpe ratio weight similar to the method from Idzorek, then divide by 2, and adjust the rest of the weights accordingly. In using this method, I simply assume that few investors are actually picking weights that optimize the Sharpe ratio. Taking half of that is a reasonable strategy. Moreover, the focus of this paper is on the potential outperformance of the active investment strategy, not necessarily the passive strategy.

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[12] For simplicities sake, I assume that the total level of T-note and T-bond debt outstanding is the value of the market capitalization for fixed income. This choice is largely driven by the fact that these weights come close to approximating a 60/40 allocation between U.S. stocks and bonds when evaluated in isolation.

The next strategy, risk-parity portfolios, replicates an approach popularized by Panagora Asset Management[13]. They seek to estimate the amount of risk of each asset class and set them equal to each other. For instance, in a typical 60/40 stock/bond asset management allocation, the contribution to overall portfolio risk of the stock position is significantly greater than 50% since stocks are much riskier than bonds. A 40/60 stock/bond allocation would result in stocks and bonds contributing more equally to total portfolio risk. The biggest benefit of the strategy is that it is more intuitive than equal-weighted portfolios. Instead of simply using an equal-weighted portfolio, the investor puts an equal weight on the riskiness of each asset. The biggest weaknesses to risk-parity portfolios are that it pays no attention to the risk premia of each asset class or the underlying economic significance of each asset. I implement risk-parity portfolios utilizing Component Value at Risk (CVAR). CVAR is the contribution of each asset to portfolio value at risk. Using the formula in Jorian (2006) for CVAR[14], error for each asset is computed as the difference of between CVAR and 1/n, where n=5 in this case. A vector containing the error terms is then multiplied by its transpose to get squared error. Weights are adjusted until the squared error falls below a pre-chosen bound[15].

In order to overcome the downside of risk-parity portfolios that they pay no attention to the economic significance of each asset, the error variable above can be adjusted so that the instead of taking the difference between CVAR of each asset and 1/n, an investor could take the difference between CVAR and the market-cap weights. For instance, in the above example with a 60/40 stock/bond allocation, there's no a priori reason why the investor should assume equal risks for each asset[16]. Alternately, creating a new risk-parity (market-cap adjusted) portfolio that

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[13] See Qian (2005) for further details on Panagora's strategy.

[14] Component Value at Risk is calculated as $CVAR_i=w_i*\beta_i$, where $\beta=\Sigma*w/(w'*\Sigma*w)$. In these equations w represents a vector of portfolio weights, Σ represents an nXn covariance matrix, and β represents the beta of each asset.

[15] This method works particularly well with a small number of assets, but is somewhat slower when dealing with large numbers of assets. If dealing with computing the risk-parity weights of sectors or sub-indices of various asset classes, it tends to help to perform the risk-parity calculations within asset classes before computing the risk-parity calculations between asset classes when including all the sub-indices, since it is possible to use the within asset class risk-parity estimates as a first approximation.

[16] There's no a priori reason either why market-cap weights should be used. In general, this approach is much more flexible and intuitive to creating portfolio weights than some other methods. For instance, a portfolio manager could take the momentum for several U.S. sector ETFs and normalize it (call that Z). Then, she could take the market-cap weights (call it W) and create a new variable that is $M=W*(1+k*Z)$, where k is a constant. If the sum of M is greater than 1, each element of M can be divided by the sum of M. Finally, she can use M in the algorithm that calculates the weights in the risk-parity calculation. This is

is broken into 60% risk for stocks and 40% for bonds will account for the economic significance of each asset class. Again, there is still the downside that these weights do not incorporate the risk premia for each asset class. However, market-cap weights, which flow through indirectly, do incorporate to some degree historical returns and investor expectations of future returns.

Next, the minimum variance portfolio (MVP) could be used as the underlying benchmark. MVP portfolios tend to benefit by reducing the allocation to the riskiest assets. Again, they do not incorporate the returns of the underlying assets.

Finally, Sharpe ratio and mean-variance optimization can be run to calculate portfolio weights[17]. The benefit of these strategies is that they incorporate both mean returns and the covariance matrix of each asset class. Further, there is an underlying intuition in trying to create portfolios that maximize the Sharpe ratio. The mean-variance optimization can be performed using several different risk aversion coefficients, taking into account investor tolerance for risk (which none of the other strategies take into account). However, there is typically large estimation error, especially in mean-variance optimization, and the weights do not reflect economic significance of the asset classes.

Estimation error is the problem that the means and covariance matrices are estimated with an error. This can cause large swings in the portfolios as risk-appetite increases. For instance, an optimization of U.S. large-cap stocks, small-cap stocks, and long-term bonds will show very high weights for small-cap stocks as risk appetite increases because they have some of the strongest historical returns. Unfortunately there is significant error in the estimation of the returns and standard deviations. There are two main ways to deal with estimation error in portfolio creation, resampling portfolios and robust Bayesian asset allocation[18]. This paper utilizes resampling using a modified version of Michaud (1998). Resampling takes the sample mean vector and sample covariance matrix and simulates a large amount of returns many times. Efficient frontiers are calculated in each of the simulations and then the set of efficient frontiers are merged into one. Returns are simulated using the method in Meucci (2009). This paper

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likely better than simply adjusted the market-cap weights by (1+k*Z) since it can incorporate the risk of each asset better.

[17] A Sortino ratio optimization could also be performed. However, there is no a priori reason why the portfolios that maximize the Sortino optimization weights are on the efficient frontier, making it difficult to extend in Matlab. In the case of the Sortino ratio, it might be possible to adopt the heuristic approach from Estrada (2007) in performing mean-semideviation optimization to the Sortino ratio.

[18] See Meucci (2005) for further details.

departs from Michaud's by simply averaging all the efficient frontiers that are calculated to create a new one. Then, portfolio weights are chosen depending on the objective function and the portfolio returns and risk of each of these new weights[19].

There is no "Holy Grail" in picking the benchmarks. In many cases, it might just be better to evaluate the portfolio statistics of some benchmarks against each other. I report results for all the benchmarks computed in Appendices 2a and 2b. Over the period 1976 to January 2010, the Sharpe, Sortino, MAR ratios for the equal-weighted and market-cap portfolios are lower than for the respective risk-parity versions. This is because the risk-parity portfolios better incorporate diversification, despite offering lower returns. Further, market-cap portfolios typically underperform the equally weighted portfolios since the market-cap portfolios have a very small allocation to REITs and Faber's strategy has a very large allocation towards them. The minimum variance portfolio has the weakest return of the benchmarks calculated. However, since risk, on a standard deviation basis but also on a drawdown and semideviation basis, is sharply reduced, it produces the best MAR and Sortino ratios and among the highest Sharpe ratio. The benchmark that seeks to maximize the Sharpe ratio tends to fall in the middle of the pack with modest returns and risk. Even though this strategy tries to maximize the Sharpe ratio based on ex ante data, the risk-parity and minimum variance portfolios actually have the highest Sharpe ratios ex post. Interestingly enough, the resampled portfolios with objective functions that maximize mean-variance utility functions depending on risk aversion coefficients tend to perform the weakest. However, the performance improves as the risk aversion coefficient increases.

In many cases, an investment manager will be limited by the method by which they are evaluated. If a pension fund hires an investment manager that will only be evaluated on the basis of market-cap weights, then this restriction will have a large impact on the returns of the portfolio. A manager able to choose a benchmark based on the portfolio statistics of the combined passive and active portfolios might select something dramatically different than the pension fund that results in stronger overall returns to the pension fund. Since the active

[19] My method of resampling does not produce very significant differences in the Sharpe-ratio optimization weights relative to weights following an optimization without resampling.

management process in this paper employs tracking error constraints, choosing a benchmark that performs strongly is nearly as important as how to make active management decisions[20, 21].

Section 4: Active Management

The basic framework that will generate weights for the active management portfolio will be to forecast expected returns and covariances, run an optimization to generate portfolio weights, and then calculate returns and statistics for each backtest. Forecasts serving as inputs to the optimization are created using two different methods, simple conditional expected returns and standard deviations based on the 10 month moving average from Faber and a more complicated Markov-regime switching process. The optimization framework is the same regardless of what forecasts are used. Unless mentioned otherwise, trading costs are not implemented in this analysis. However, since trading only occurs once a month and since there are usually much smaller changes made to portfolios than in Faber's strategy, this will probably only minimally bias up returns. Also, all forecasts and optimizations are performed out-of-sample following a period of roughly 5 years where all the data is available to allow sufficient data to construct forecasts.

Part A: Forecasting Returns and Standard Deviations

The simplest approach to create forecasts for the expected returns and standard deviations based on the paper from Faber (2009) is to use the historical means and standard deviations conditional on whether the market is above the 10 month moving average or below. For instance, the average returns and standard deviations can be calculated for each asset when they are above the 10 month moving average and below. If the asset is above the 10 month average at the end of the most recent period, the historical average up to that point is used as the input to the optimization for the next period. In this fashion, the strategy will only use the available data and not have any forward-looking bias. For the purposes of creating the covariance matrix, I assume the standard deviations change between above the 10 month and

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[20] Some TAA managers will utilize a 60/40 benchmark for U.S. stocks and bonds. However, so long as the strategy is investing outside those traditional classes, they will show relatively strong returns or lower risk. The manager may wish to be paid relative to a 60/40 benchmark since otherwise that is how the institution or investor would have allocated funds, but It is less important for actually constructing a portfolio.

[21] I find that the Sharpe ratio maximization or market-cap adjusted risk-parity portfolios have the strongest a priori justification.

below the 10 month, but the correlations do not[22]. This is merely a simplification. It is not as intuitive when looking at the returns of one asset conditional whether the other is above or below an average. There would also need to be four conditional correlations calculated for each asset[23] which can suffer from errors upon estimation. The benefit of this method is that it is simple and easy to implement. The downside is that it is almost too simple and ignores things like how far the asset is from the moving average or other factors that may be relevant in improving the forecasts. However, future research can investigate other adjustments to improve performance. For the purposes of this paper, I wanted to keep the strategy close to Faber's.

Another method for forecasting expected returns and a covariance matrix is to estimate a multivariate Markov regime-switching equation in each period and using the conditional expected returns and standard deviations as the forecasts for the next period. Estimating Markov regime-switching equations has been a popular line of research since the seminal paper by Hamilton (1989) on the regimes and GDP growth. A regression with Markov regime-switching assumes there are k regimes (in my case I stick with two regimes throughout) and there is a probability that the coefficients in the regression will transition between regimes. For instance, Hamilton details a regime that has expansions in GDP and recessions. In one, GDP growth might rise 3% per year on average, but in the other growth will be -1% per year (with higher volatility). Various authors have extended this technique to help explain multiple regimes in the returns for stocks, bonds, exchange rates, and risk-free securities[24]. Ang and Bakaert (2002) provide a framework for forecasting returns and a covariance matrix with multiple regimes that can serve as inputs into an optimization. They estimate a multivariate Markov regime-switching model in the following form:

$$r_t = \mu * s_t + \beta(s_t) * r_{t-1}^f + \varepsilon_t$$

Where $r_t = (r_t^f, r_t^e, r_t^b)'$, $\mu = (\mu_f, \mu_e, \mu_b)'$, $B = (\rho, \beta^b, \beta^e)'$, and $\varepsilon_t = (\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3)' \sim N(0, \Lambda(s_t))$. s_t is the state of the regime, $\beta(s_t)$ is dependent on the state of the regime, $\Lambda(s_t)$ is a covariance

[22] Standard deviations calculated here are not calculated assuming different weights, though correlation coefficients do assume a weighted structure. For instance, to obtain each element of the covariance matrix, the weighted correlation matrix elements are multiplied by the relevant conditional standard deviations.
[23] Both asset classes above their respective moving averages, both below, and then two where one is above and the other one is below.
[24] See Norden and Schaller (1997), Gray (2002), and Bollen, Gray, and Whaley (2000) for more information.

matrix dependent on the regime, r_t^f is returns for a risk-free security, r_t^e is excess returns[25] for equities, and r_t^b is excess returns for bonds. They model transition probabilities as a time-varying function of the risk-free rate. They also estimate an alternate specification where the B_e=B_b=0.

I utilize the MS_Regress program available for Matlab[26] in order to estimate Markov regime-switching equations. This program does not utilize the GM algorithm and cannot implement the time-varying transition probabilities that Ang and Bakaert use. Hence, the structure of transition probabilities is constant over time[27]. Further, extending the Ang and Bakaert framework to five asset classes instead of two greatly expands the number of parameters required. This is especially true when the correlations regime-switch, but even ignoring correlations in a multivariate estimation precludes computing out-of-sample estimates in a reasonable period of time. Hence, I ultimately estimate equations of the form:

$$r_t = \mu * s_t + \beta * s_t * r_{t-1}^f + \varepsilon_t$$

Where $r_t = (r_t^f, r_t^a)'$, $\mu = (\mu_f, \mu_a)'$, $B = (\rho, \beta^a)'$, and $\varepsilon_t = (\varepsilon_t^1, \varepsilon_t^2)' \sim N(0, \Lambda(s_t))$. s_t is the state of the regime[28], $\Lambda(s_t)$ is a covariance matrix dependent on the regime, r_t^f is the returns of risk-free securities, r_t^a is excess returns for an individual asset class. The above two-variable system is then estimated using only the available data in each time period for each asset class so long as at least 5 years of returns are available.

Each conditional variable in time t is used as the best forecast for time t+1. For each asset class the conditional forecast of expected excess returns is then added to the risk-free rate in each case to obtain the conditional expected returns for the asset class. The conditional standard deviations can be calculated for each asset class. The covariance matrix is generated utilizing the same method as with the moving average forecasts[29].

Part B: Optimization

After generating the expected return and covariance matrix forecasts, the next step in portfolio construction is to run an optimization to obtain active portfolio weights. Da Silva et al.

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[25] Excess returns are calculated as the actual total returns minus the risk-free returns.
[26] Perlin (2010)
[2727] This just means that if the probability of switching from a bull regime to a bear regime is x%, then this is constant over time and will not change. However, in any given time, the conditional probabilities are not constant.
[28] Assuming two regimes.
[29] The weighted correlation matrix is calculated in each period and then it is multiplied by the necessary standard deviations to obtain the covariance matrix.

(2009) develop a Black-Litterman model[30] for use in active management. Essentially, they set the equilibrium returns to 0 and simplify the equation for Black-Litterman returns to one that can be used easily for active management:

$Max_{wa}\ w_a'\mu - .5*\lambda*w_a'\Sigma w_a$, s.t. other constraints

Where w_a represents a vector of active weights, Σ is the covariance matrix, λ is a risk aversion coefficient, and after simplification $\mu = 0.5 * \Sigma P'*(P\Sigma P')^{-1}*Q$. In this case P represents the view matrix in the Black-Litterman methodology and Q is a vector of returns for each view. This formula is effective since the weights in the benchmark do not have to be equivalent to the market-cap weights. Relative to the original Black-Litterman framework, it provides enough flexibility so that any benchmark could be used. However, while the formula can be effective if the investor states views on only a small number of the total number of securities, if she states a view on every security, then the P matrix becomes an identity matrix and the formula for μ reduces to

$\mu = 0.5 * \Sigma*\Sigma^{-1}*Q = 0.5 * Q$

Since in an optimization it does not matter if the returns are divided in half, the optimization boils down to an active returns-tracking error optimization of the form:

$Max_{wa}\ w_a'\mu - .5*\lambda*w_a'\Sigma w_a$, s.t. other constraints

Where w_a represents a vector of active weights, Σ is the forecasted covariance matrix, λ is a risk aversion coefficient, and μ equals forecasted returns. I make additional adjustments to include the cost of leveraged returns and utilize constraints so that $(w_a'\Sigma w_a)^{.5} < B$, where B is a tracking error bound, $0 <= w_a*1 <= L$, where L is a maximum leverage bound, and $w_a >= -w_b$, where w_b is the weight in the benchmark[31].

Active portfolio weights were calculated using the above optimization framework in each period where forecasts for expected returns and covariance matrices are available. Several different coefficients for risk aversion, tracking error bounds, and maximum leverage are tested, holding them constant through every optimization.

Part C: Results

Active weights from the optimization were then added to the benchmark weights to create final portfolio weights. Returns and summary statistics were then computed for each

[30] See Walters (2009) for a good explanation of the Black-Litterman methodology.
[31] This is a long-only constraint.

portfolio. Results are available in Appendices 3a and 3b[32]. Using conditional expected returns and standard deviations based on the 10 month moving average generally results in positive information ratios, smaller drawdowns, and lower semideviations relative to the benchmarks. Markov regime-switching coefficient expected returns and standard deviations show similar results, but actually tend to underperform the simple moving average strategy[33]. One reason why this could be is that the forecasted returns in the moving average section are quite stable, except when they make a sharp switch. However, the Markov regime-switching estimates are less stable over time due to the changing estimates of conditional probabilities of the state of the regime. This actually leads to more trading than would happen in the moving average strategy. Further, the Markov switching strategy does not necessarily assume that the regimes are significant in each period, but the moving average strategy does. The Markov strategy avoids any forward-looking bias that the moving average strategy might have.

Results are reported in the appendices with a base case where the maximum leverage allowable is 25%, the risk aversion coefficient is 3, and the tracking error bound is 3%. For this case, both the moving average and Markov regime-switching forecasts result in positive Jensen's alpha against the benchmark and higher Sharpe/MAR/Sortino ratios than their respective benchmarks. The simpler moving average strategy tends to outperform the more complex Markov regime-switching strategy on virtually every ratio. Ex post, the tracking error relative to the benchmark averages 2.5% for the moving average strategy and 3% in the Markov regime-switching strategy[34]. This is a much narrower tracking error than Faber's strategy. The biggest difference in weights tends to be that the risk-free rate gets very little allocation in this strategy. Most of the weights that would have moved to risk-free securities in Faber's strategy tend to switch to U.S. bonds in either of these. Further, the strategy is reluctant to reduce leverage when the cost of borrowing is low.

[32] Full results are available upon request. Appendices only report maximum leverage of 25%, risk aversion coefficient of 3, tracking error bound of 3% for the sake of space.

[33] The Markov regime-switching forecasted returns have a 20-30% correlation on average with their simple alternatives. At best, the correlation is 51% for U.S. stocks. At worst, there is only 2.6% correlation for 10-year bonds.

[34] And 7% for the original Faber strategy.

Ceteris paribus, increasing the leverage from 0% to 25% results in higher alphas, Sharpe ratios, MAR ratios, and Sortino ratios[35] However, these values are only modestly higher when moving from 25% to 50% due to the increase in risk. Similarly, increasing the risk aversion coefficient from 1 to 3 to 5 results in lower Sharpe/MAR/Sortino ratios and less alpha[36]. Finally, moving from a 1% tracking error bound to a 3% tracking error bound results in a significant improvement in alpha and these ratios. However, moving from a 3% bound to a 5% bound makes little impact since the bound becomes less of a constraint.

Since the beta of this strategy is nearly always above 1 whereas Faber's strategy is usually substantially below 1 and since allocations towards risk-free securities are much less aggressive than Faber's strategy, the optimizations using moving average and Markov regime-switching expected returns and standard deviations tend to match declines in the benchmark much more closely than Faber's strategy does. This is clearly visible in the equity curves in Appendices 6a and 6b. For instance, over period from December 2007 through February 2009 when markets corrected sharply, these strategies track the benchmark much more closely than Faber's strategy, despite having large active weights on bonds[37]. The underperformance during the crisis mainly results from leverage held near the maximum allowable amount instead of adjusting lower[38]. The overall portfolio is positioned more defensibly, but since the portfolio stays levered near the maximum amount when the cost of borrowing is low, it merely matches the market declines. While reducing tracking error would be more attractive to many financial institutions, it also means that the strategy will still decline sharply during market corrections.

Section 5: Extensions

Part A: Adjusting Maximum Leverage, Tracking Error Bounds, and Risk Aversion Coefficients

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[35] I don't include a component so that borrowing is more costly than investing in the risk-free rate, so this result would probably be much weaker.

[36] This is different than the finding from the benchmark, possibly due to the adjustment I made to active risk in the objective function so that active risk and active returns were more comparable. Otherwise the risk aversion coefficient was having no effect whatsoever.

[37] This is merely for illustrative purposes. Faber's strategy there does not use leverage, while the others do, so this may not be the best comparison.

[38] I also investigated whether increasing the cost of borrowing by 1% over the risk-free rate would reduce leverage. My preliminary investigation revealed that it wouldn't have made much of an impact over the recent crisis since rates are so low, but when the absolute cost of borrowing is higher it can make a difference.

The benefit of the framework presented in the previous section is that it can be more flexible in making adjustments. For instance, the tracking error bounds, maximum leverage, and risk-aversion coefficients were held constant in the previous section. However, if investors believe that returns on various asset classes are cyclical, there is no a priori reason why they want to keep tracking error bounds, maximum leverage, and risk-aversion constant over time. The biggest benefit to adjusting these coefficients is that performance can be improved in a relatively simple manner.

To adjust these coefficients, I adopted the following generic form:

$X=Max(0,X_{base}+k*Z)$

Where X_{base} equals an average level of each variable, k equals $(X_{max}-X_{base})$[39], and Z is a variable normalized between -1 and 1.

There are several different methods to construct the Z variables. Two of the most simple might be to use the active management returns and the benchmark returns. For instance, if the benchmark returns are strong relative to the available history, should the manager increase maximum leverage or increase risk-aversion coefficients. These are essentially equivalent to a trader adjusting his strategy based on their equity curve, except that it is possible to separate out the benchmark and the active strategies.

Another possible Z variable is the Baa yield-10 year Treasury yield spread. Generally recessions and stock market corrections are associated with higher levels of this spread as the cost to borrow funds for businesses increases due to rising probabilities of default and losses given default. For instance, it might be profitable for an investor to reduce leverage or risk aversion coefficients when the Baa spread increases.

Finally, the probability of being in one regime or another from the Markov regime-switching model can be normalized into a Z variable. This is similar to the Baa spread above. If there is an increasing probability that the market is going to be in a low expected return regime, then an investor might decrease the maximum amount of leverage in the portfolio. This calculation becomes difficult out-of-sample since the estimates for conditional probabilities at time t are not from the same estimation as for time t+1. Also, this is easier to implement with a multivariate framework but less intuitive with five asset classes where a regime in one asset class may not mean the same as a regime in another.

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[39] X_{max} is the maximum tolerable value of X, though it could also be specified in terms of X_{min}. The reason for this choice of k is so that X will linearly travel between X_{min} and X_{max} depending on the Z variable.

The strongest improvements to returns come from making adjustments to maximum leverage, risk aversion coefficients, and tracking error based on the Baa-10-year Treasury spread. There are also modest improvements to Sharpe/MAR/Sortino ratios by making adjustments based on active and passive portfolio returns, though less of an improvement in alpha.

Combining all the variables together[40] produces some of the most consistently solid returns using a variety of benchmarks. Results for the combined strategy are in Appendix 4a and 4b. Whereas some of these strategies might work best with one type of benchmark (like equally weighted ones), the combined statistics are more robust to different types of benchmarks, resulting in improvements to Sharpe/MAR/Sortino ratios across benchmarks. Furthermore, the strongest improvements tend to occur in the benchmarks that had the strongest performance in these ratios, like equally weighted portfolios and minimum variance portfolios.

Over long periods of time, the impact of these changes might be relatively minor. However, there can be substantial improvements relative to the basic strategy during market corrections. During the economic correction from December 2007 through March 2009, this extension helped reduce drawdowns substantially relative to the basic optimizations in the previous sections. However, the rebound from the recovery is weaker. Beyond the changes made in this section, it will be worthwhile to examine other strategies to decrease leverage and increase holdings of risk-free securities during recessions if clients seek some of the protection from market corrections that Faber's strategy provides.

Part B: Breaking Asset Classes into Sub-indices

Another extension of this framework is to break each asset class into sub-indices. For instance, U.S. stocks could be broken into sectors, foreign stocks broken into countries, etc. There are multiple methods to forecast returns for each of the sub-indices, but this paper focuses solely on the cyclicality and idiosyncratic momentum of the individual sub-indices. Richer models could also be used, but this framework for simplicity and underlying economic intuition.

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[40] I.e. averaging each of the three scaled variables created from active returns, passive returns, and the Baa spread. These create a combined maximum leverage, risk aversion, and tracking error variables that change over time. The results in this section refer to incorporating all three of the maximum leverage, risk aversion, and tracking error variables in one optimization. Rather than in isolation like the previous paragraph does.

Foreign stocks are broken down into Australia, Canada, France, Germany, Hong Kong, Italy, Japan, Netherlands, New Zealand, Singapore, Spain, Sweden, Switzerland, and the U.K. U.S. stocks are broken into the 10 S&P500 industries[41]. For commodities, the Energy, Industrial Metals, Precious Metals, Livestock, and Agriculture sub-indices from the GSCI are used. For bonds, the U.S., U.K., Japan, France, Germany, Italy, and Spain traded GBI indices from J.P. Morgan.[42] Finally, no sub-index for REITs is used. Like the vanilla implementations in Section 4, all returns are in U.S. dollars[43].

Since forecasted returns for each asset class were generating in Section 4, the forecast of each sub-index relative to their benchmark is the most intuitive approach. For each data point, the excess returns of the sub-index are regressed against the excess benchmark returns[44]. Residuals are collected and then averaged to find the past 6 months average error[45]. I make the assumption that the average value of the past 6 months average error will continue into the next period. This can be thought of as an idiosyncratic momentum factor. Using the alpha and beta computed in the regression, plus the benchmark return will generate a simple output for excess returns of the sub-index. By incorporating the beta of each asset class relative to its benchmark, the estimates can incorporate cyclicality. Adding in the idiosyncratic momentum factor to the simple estimates can help account for the momentum of the individual indices or other factors driving returns not well captured by the regression[46].

The biggest roadblock to breaking down the returns to individual sub-indices is that the returns may not be available over the whole period that the benchmark is available and that the market-cap weights for the sub-indices are not always available. For instance, several energy futures contracts were not available until the early 1980s and so the Energy GSCI index is not available until the early 1980s. Also, data for foreign country bond returns are also not available

[41] For reference, that is Consumer Discretionary, Consumer Staples, Energy, Financials, Healthcare, Industrials, Information Technology, Materials, Telecomm, Utilities

[42] This is the only group of sub-indices where one of the sub-indices replacing the original asset class is functionally equivalent to the original asset class.

[43] This provides another room for improvement in the strategy. Incorporating currency momentum or some other forecasts of currency returns could add significant value.

[44] I wait until five years worth of data are available before beginning the regressions. Regressions are performed at each data point with all available data up to that point. As an alternative, I considered using rolling 5 year windows for the regressions. Results are similar.

[45] I chose 6 months for simplicity, but there's no reason why different lengths can be chosen. I made no effort to attempt to find the best length.

[46] Returns are also calculated assuming the average error/idiosyncratic momentum is equal to 0. Results are available upon request.

until the mid-1980s. I ultimately chose to not begin this exercise until sufficient data was available so that calculations could be made for each asset class (the strategy does not begin until the early 1990s). Finally, some of the MSCI country indices do not go back to the start of the MSCI EAFE index and there have been several different countries added or subtracted from the EAFE index[47]. Hence, for these indices, I ignore countries where I do not have sufficient data to either create returns forecasts or obtain portfolio weights[48] until there is sufficient data. These issues make it very difficult to construct weights for component sub-indices such that when combined together they match the original benchmarks calculated in Section 3. Hence, if beta is calculated for the active part of this strategy, then there will be higher alphas when regressed against the original benchmarks than there would be regressed against a more accurate benchmark. Sharpe/MAR/Sortino ratios are probably the best method to compare the different strategies.

Further, in order to create benchmark portfolios similar to section 3, weights are required for the sub-indices. I generally follow the same strategy to create weights for the sub-indices as I did with the indices. For example for the equally-weighted benchmark, U.S. as a whole will be equally weighted with other asset classes and each U.S. sector will be equally weighted relative to each other. Creating market-cap weights for commodities sub-indices is perhaps the biggest challenge. Bloomberg has available the historical GSCI weights going back to 2004, but does not provide the data going back further. I make the assumption that the weights of the sub-indices in the GSCI index multiplied by the market-cap estimated in section 3 are equivalent to the market-cap weights. In order to estimate the weights for commodities in the earlier periods, I assumed that an investor was a mean-variance optimizer who wanted Beta=1 exposure to the GSCI commodities index in each period using the various sub-indices. I compared this to the actual weights and found them comparable. Estimated weights are only used prior to when I have access to the actual historical weights. When creating the minimum variance, Sharpe ratio maximization, and mean-variance optimization weights for the new benchmark portfolios, I simply assumed the weights for the sub-indices were market-cap weights[49], but adjusted so they would equal the index weights calculated for their respective benchmarks in section 4.

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[47] I also include Canada which has never been a member of the MSCI EAFE index.
[48] The former is typically the limiting factor.
[49] The motivation for this was purely due to time constraints.

Results are reported from late 1994 to January 2010 in Appendices 5a and 5b. The biggest benefit to using the sub-indices is that during different parts of the business cycle, the sub-indices can perform quite differently. For instance, U.S. sectors with lower betas will benefit during recessions relative to sectors with higher betas. In general, the results from this exercise show a strong (albeit biased) Jensen's alpha from taking advantage of sectors. The strongest improvements come from Sharpe and Sortino ratios, but the MAR ratios only modestly increase. While this could be due to an over-exposure of cyclical assets right before the 10 month moving average switches from below prices to above prices, the Markov regime-switching strategy also tends to be overweight cyclical assets during economic expansions. Averaging across all of the benchmarks, using moving average conditional expected returns and standard deviations but also takes into account sub-indices adds 0.23 to the Sharpe ratio, a 0.40 to the Sortino ratio, and 0.08 to the MAR ratio, relative to the strategy that only takes into account conditional moving average expected returns and standard deviations at the asset class level only. Betas also tend to average close to 1 in this strategy relative to the benchmarks from section 3, making them more attractive to institutions who are interested in less tracking error from the benchmark. Suffice it to say, though this strategy excludes trading costs, the improvement in risk-adjusted returns generated from paying attention to regimes and how regimes impact both asset classes and their sub-indices is very important[50].

Section 6: Conclusion

Faber provides a simple strategy to reduce the risk of implementing an asset allocation strategy. However, the strategy may not coincide with the investment goals of financial institutions due to the binary investment decisions used. The advantages of investment strategies dependent on the regimes of different asset classes are important enough that institutions should not avoid TAA. Small changes tweaking Faber's strategy can facilitate making investments on the basis of regimes more palatable to financial institutions. Furthermore, by using these improvements an active manager implementing extensions to the strategy can improve returns or limit risk.

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[50] I also calculated what happens when combining the sub-index strategy with the strategy with the strategies that adjusts coefficients for maximum leverage, risk aversion, and tracking error. Preliminary investigations reveal little value-added relative to the sub-index strategy over this period where data is available by adjusting the coefficients. However, it seems that this is entirely due to a drag from the risk aversion coefficients. It could be that what was a good relationship for risk aversion in the early years reversed.

Future research can center on adjusting the parameters of the optimization to find more effective ways to push up the risk-free security allocations or decrease the maximum leverage components more effectively during market corrections in order to more closely replicate Faber's strategy.

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Appendix 1: Data Sources

Unless mentioned otherwise price, total return, and market capitalization data are all obtained from the same location. All series are denominated in U.S. dollars.

S&P500 – Factset

MSCI EAFE – Factset

10-year Treasury Total Return – Global Financial Database

10-year Treasury estimated market cap – the sum of Treasury bond and Treasury note outstanding debt - Factset

GSCI Index – Bloomberg

NAREIT Index – National Association of Real Estate Investment Trusts (prior to march 2006, market capitalization estimates had to be interpolated from annual data)

3-month T-bill (secondary market) yield - St. Louis Federal Reserve

10-year constant maturity Treasury yield - St. Louis Federal Reserve

Moody's Baa corporate yield – St. Louis Federal Reserve

S&P 500 industry breakdown (Consumer Discretionary, Consumer Staples, Energy, Financials, Healthcare, Industrials, Information Technology, Materials, Telecomm, Utilities) – Factset

MSCI Countries (Australia, Canada, France, Germany, Hong Kong, Italy, Japan, Netherlands, New Zealand, Singapore, Spain, Sweden, Switzerland, and the U.K.) – Factset

GSCI Sub-indices (Energy, Industrial Metals, Precious Metals, Livestock, and Agriculture) – Bloomberg

J.P. Morgan GBI traded indices (U.S., U.K., Japan, France, Germany, Italy, and Spain) – J.P. Morgan/Morgan Markets

Appendix 2a: Passive portfolios (Jan. 1977 - Jan. 2010)

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.009	0.009	0.008	0.008	0.008	0.008	0.008	0.008	0.008
standard deviation	0.029	0.034	0.024	0.027	0.021	0.030	0.040	0.037	0.033
sharpe ratio	0.473	0.360	0.500	0.427	0.452	0.379	0.198	0.234	0.338
MAR ratio	0.228	0.210	0.287	0.279	0.550	0.304	0.148	0.179	0.241
sortino ratio	0.720	0.553	0.807	0.694	0.811	0.616	0.294	0.352	0.530
observations	396	396	396	396	396	396	396	396	396

Appendix 2b: Passive portfolios (Sept. 1994 - Jan. 2010)

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.007	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.006
standard deviation	0.032	0.036	0.026	0.028	0.020	0.027	0.043	0.039	0.033
sharpe ratio	0.359	0.208	0.465	0.330	0.574	0.341	0.096	0.136	0.239
MAR ratio	0.163	0.129	0.220	0.192	0.453	0.211	0.086	0.109	0.157
sortino ratio	0.495	0.288	0.669	0.474	0.921	0.496	0.130	0.185	0.337
observations	184	184	184	184	184	184	184	184	184

Weq = equally weighted portfolio, wmktcp = market-cap weights, wrp = risk-parity weights, wrpmktcp = risk-parity (market-cap adjusted) weights, wmin = minimum variance weights, wrsmplshp = Sharpe ratio maximization weights, wrsmplMVO1-3 equals mean variance optimization weights with risk aversion coefficients from 1 to 3 to 5. mean and standard deviation are calculated on a monthly basis, all other variables are calculated based on the annualized data.

Appendix 3a: Portfolios Constructed with Moving Average Inputs

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.011	0.011	0.010	0.010	0.011	0.010	0.010	0.011
standard deviation	0.034	0.038	0.029	0.031	0.025	0.035	0.045	0.043	0.039
sharpe ratio	0.667	0.519	0.715	0.610	0.702	0.561	0.327	0.376	0.500
MAR ratio	0.284	0.259	0.367	0.351	0.676	0.387	0.183	0.221	0.304
sortino ratio	1.039	0.814	1.185	1.017	1.293	0.941	0.497	0.582	0.813
information ratio	0.322	0.299	0.322	0.305	0.376	0.346	0.292	0.296	0.327
beta	1.148	1.122	1.176	1.156	1.160	1.171	1.125	1.148	1.160
alpha	0.024	0.022	0.023	0.021	0.022	0.023	0.020	0.021	0.022
observations	396	396	396	396	396	396	396	396	396

Appendix 3b: Portfolios Constructed with Markov Regime-switching Inputs

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.010	0.010	0.010	0.009	0.010	0.010	0.010	0.010
standard deviation	0.035	0.039	0.031	0.033	0.026	0.037	0.046	0.044	0.040
sharpe ratio	0.610	0.464	0.628	0.537	0.596	0.510	0.327	0.365	0.467
MAR ratio	0.270	0.244	0.328	0.307	0.513	0.344	0.183	0.216	0.283
sortino ratio	0.935	0.712	1.003	0.860	1.038	0.830	0.493	0.558	0.738
information ratio	0.232	0.212	0.216	0.208	0.217	0.248	0.255	0.253	0.253
beta	1.177	1.154	1.220	1.207	1.201	1.216	1.151	1.170	1.197
alpha	0.018	0.015	0.015	0.014	0.015	0.017	0.021	0.020	0.018
observations	396	396	396	396	396	396	396	396	396

Portfolios are constructed with maximum leverage = 25%, risk aversion coefficient = 3, tracking error bound = 3%. Weq = equally weighted portfolio, wmktcp = market-cap weights, wrp = risk-parity weights, wrpmktcp = risk-parity (market-cap adjusted) weights, wmin = minimum variance weights, wrsmplshp = Sharpe ratio maximization weights, wrsmplMVO1-3 equals mean variance optimization weights with risk aversion coefficients from 1 to 3 to 5. mean and standard deviation are calculated on a monthly basis, all other variables are calculated based on the annualized data.

Appendix 4a: Portfolios Constructed with Moving Average Inputs and Scaling Factors

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.011	0.011	0.010	0.010	0.011	0.010	0.010	0.011
standard deviation	0.032	0.037	0.028	0.031	0.024	0.035	0.044	0.041	0.038
sharpe ratio	0.706	0.537	0.752	0.620	0.722	0.574	0.339	0.406	0.517
MAR ratio	0.335	0.286	0.466	0.395	0.910	0.482	0.193	0.257	0.361
sortino ratio	1.148	0.856	1.298	1.052	1.385	0.984	0.522	0.644	0.860
information ratio	0.277	0.297	0.288	0.291	0.333	0.312	0.276	0.301	0.302
beta	1.070	1.082	1.104	1.120	1.105	1.136	1.096	1.107	1.124
alpha	0.028	0.023	0.027	0.021	0.024	0.024	0.022	0.025	0.024
observations	396	396	396	396	396	396	396	396	396

Appendix 4b: Portfolios Constructed with Markov Regime-switching Inputs and Scaling Factors

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.011	0.010	0.010	0.009	0.010	0.010	0.010	0.011
standard deviation	0.035	0.039	0.031	0.033	0.026	0.038	0.046	0.044	0.041
sharpe ratio	0.616	0.489	0.629	0.546	0.597	0.498	0.348	0.380	0.461
MAR ratio	0.273	0.262	0.333	0.326	0.540	0.341	0.194	0.228	0.281
sortino ratio	0.946	0.767	1.011	0.889	1.055	0.809	0.533	0.586	0.728
information ratio	0.207	0.215	0.197	0.196	0.209	0.211	0.244	0.238	0.220
beta	1.177	1.137	1.221	1.207	1.196	1.232	1.140	1.165	1.210
alpha	0.020	0.019	0.017	0.015	0.015	0.017	0.024	0.023	0.018
observations	396	396	396	396	396	396	396	396	396

Portfolios are constructed with a base maximum leverage = 25%, risk aversion coefficient = 3, tracking error bound = 3%. However, these will adjust on the basis of the average of three scaled variables: prior benchmark returns, prior active returns, and the Baa-Treasury spread. Weq = equally weighted portfolio, wmktcp = market-cap weights, wrp = risk-parity weights, wrpmktcp = risk-parity (market-cap adjusted) weights, wmin = minimum variance weights, wrsmplshp = Sharpe ratio maximization weights, wrsmplMVO1-3 equals mean variance optimization weights with risk aversion coefficients from 1 to 3 to 5. mean and standard deviation are calculated on a monthly basis, all other variables are calculated based on the annualized data.

Appendix 5a: Portfolios Constructed with Moving Average Inputs and Sub-index Breakdowns

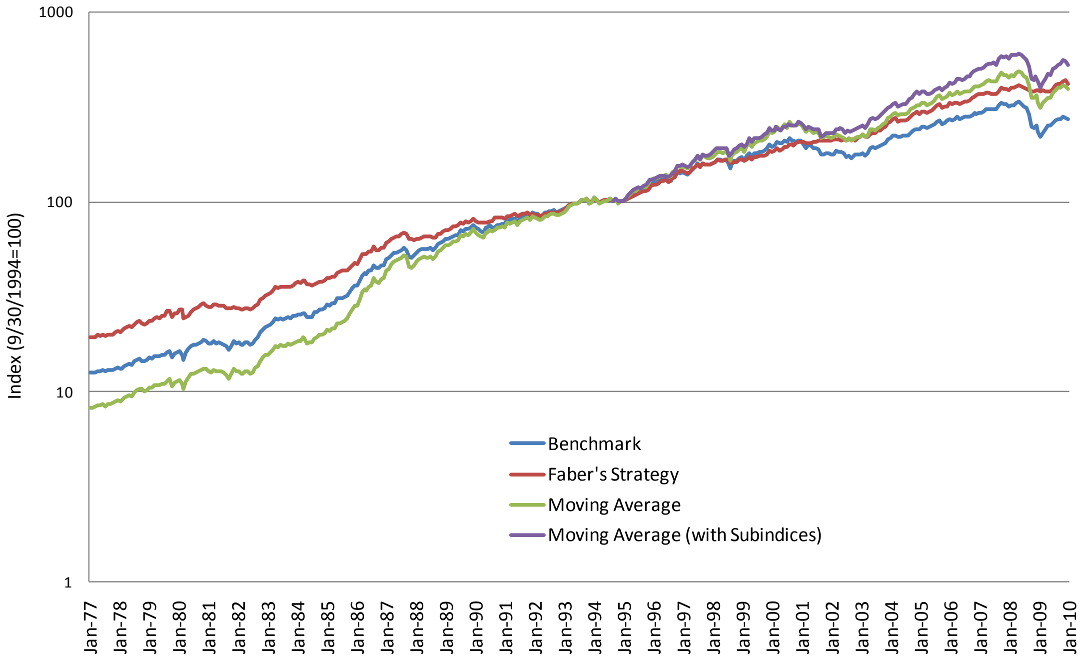
	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.009	0.010	0.010	0.009	0.009	0.010	0.009	0.009
standard deviation	0.038	0.037	0.032	0.031	0.026	0.033	0.047	0.044	0.039
sharpe ratio	0.671	0.562	0.823	0.729	0.842	0.666	0.435	0.435	0.540
MAR ratio	0.263	0.251	0.338	0.338	0.455	0.325	0.189	0.198	0.251
sortino ratio	1.001	0.845	1.280	1.141	1.445	1.033	0.620	0.615	0.792
information ratio	0.270	0.260	0.249	0.320	0.151	0.192	0.176	0.171	0.182
beta	1.115	0.944	1.085	1.042	0.884	1.004	0.917	0.965	1.003
alpha	0.045	0.047	0.047	0.046	0.040	0.044	0.058	0.048	0.045
observations	184	184	184	184	184	184	184	184	184

Appendix 5b: Portfolios Constructed with Markov Regime-switching Inputs and Sub-index Breakdowns

	weq	wmktcp	wrp	wrpmktcp	wmin	wrsmplshp	wrsmplMVO1	wrsmplMVO2	wrsmplMVO3
mean	0.011	0.009	0.010	0.010	0.009	0.009	0.010	0.009	0.009
standard deviation	0.038	0.037	0.032	0.031	0.026	0.034	0.047	0.044	0.040
sharpe ratio	0.678	0.554	0.806	0.738	0.828	0.635	0.427	0.430	0.528
MAR ratio	0.279	0.254	0.353	0.362	0.478	0.327	0.190	0.202	0.259
sortino ratio	0.999	0.830	1.230	1.162	1.400	0.955	0.599	0.598	0.760
information ratio	0.272	0.246	0.247	0.317	0.150	0.184	0.173	0.170	0.180
beta	1.111	0.938	1.105	1.027	0.902	1.033	0.922	0.975	1.020
alpha	0.046	0.046	0.045	0.046	0.039	0.041	0.057	0.048	0.044
observations	184	184	184	184	184	184	184	184	184

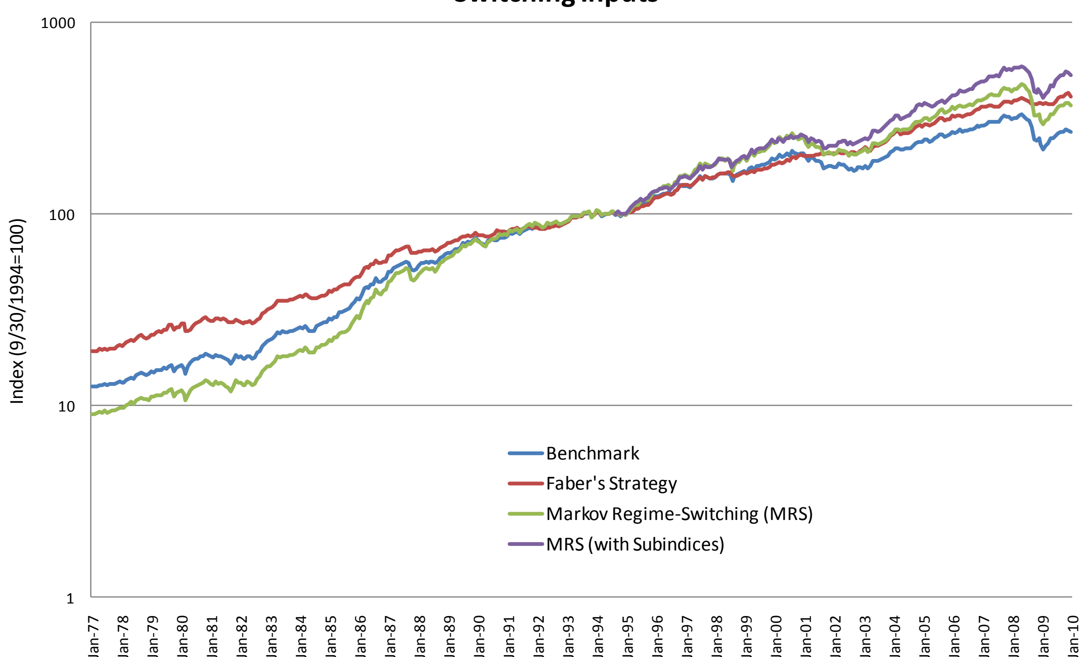
Each of these strategies breaks down each asset class into sub-indices. Portfolios are constructed with maximum leverage = 25%, risk aversion coefficient = 3, tracking error bound = 3%. Weq = equally weighted portfolio, wmktcp = market-cap weights, wrp = risk-parity weights, wrpmktcp = risk-parity (market-cap adjusted) weights, wmin = minimum variance weights, wrsmplshp = Sharpe ratio maximization weights, wrsmplMVO1-3 equals mean variance optimization weights with risk aversion coefficients from 1 to 3 to 5. mean and standard deviation are calculated on a monthly basis, all other variables are calculated based on the annualized data.

Appendix 6a: Equity Curves for Optimizations with Moving Average Inputs



Note: the MA strategies here are the same as those reported in the earlier Appendices with a benchmark composed of risk-parity (market-cap adjusted) weights. Hence, they incorporate leverage, whereas the benchmark and Faber's strategy do not.

Appendix 6b: Equity Curves For Optimizations With Markov Regime-Switching Inputs



Note: the MRS strategies here are the same as those reported in the earlier Appendices with a benchmark composed of risk-parity (market-cap adjusted) weights .Hence, they incorporate leverage, whereas the benchmark and Faber's strategy do not.